THE UNIVERSITY OF ALBERTA

VIOLATIONS OF CP SYMMETRY IN WEAK INTERACTION

by

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A THESIS

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ABSTRACT

A brief survey of the various suggestions as to the origin of CP noninvariance effects is made. After a brief study of the method of chiral dynamics in SU(2)xSU(2) group, a chiral first class current-current phenomenological Lagrangian Model for weak interactions is proposed. CP violation effects are attributed to the interference between $\Delta I = 1/2$ and $3/2$ channels. The amplitudes for kaon to two-pion decays are calculated using PCAC and the static approximation for the π-π phase shift. A discrepancy at two standard deviations in the branching ratio of K_S to 2π suggests the requirement of some contribution from the $\Delta I = 5/2$ transition. Predicted CP violation parameters agree with world averaged experimental data, but are slightly higher than the most recent reported values. This discrepancy, together with the observed charge asymmetry in semi-lepton decay of K_L could be reconciled by adding a $\Delta S = 2$ term into our Lagrangian. The possibility of CP violation in baryon reactions is also discussed.

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CONTENTS

CHAPTER I INTRODUCTION

Since the discovery by Christenson et al[1] that the long-lived component K_L of the $K^0 - \bar{K}^0$ system decays into two charged pions,

$$K_L \rightarrow \pi^+ + \pi^- \qquad (1)$$

which is disallowed if CP parity is conserved during decay processes, much effort has been dedicated to the search for other effects which may confirm the violation of CP invariance in $K^0 - \bar{K}^0$ decay, and many suggestions have been made as to the origin of these CP noninvariant effects.

The phenomenological description of the $K^0 - \bar{K}^0$ system as a degenerate system with degenercy removed by weak interactions, was first made by Gell-Mann & Pais[2] , and subsequently developed by Treiman and Sachs[3]. The existence of two linear combinations of K^0 and $\bar{K}^0$ states with slightly different mass and life time was predicted and experimentally verified[4]. The main decay modes of the short and long lived neutral kaon lead to two pions and three pions, semileptonic respectively.

The CP parity of the two- and three-pion states are determined as follows:

Operating on the two-pion system with CP gives

$$
\begin{aligned}
CP \mid \pi^+ \pi^- > &= \xi^P_{\pi^+} \xi^P_{\pi^-} (-1)^L C \mid \pi^+ \pi^- > \\
&= (-1)^2(-1)^L \mid \pi^- \pi^+ > \\
&= (-1)^L(-1)^L \mid \pi^+ \pi^- > \\
&= + \mid \pi^+ \pi^- >,
\end{aligned} \tag{2}
$$

The last relation arises because the charge conjugation operation simply exchanges the charges of the two pions, which is equivalent, on the validity of Bose statistics, to an exchange of spatial coordinates. (The two pions states would not be an eigenstate of CP if pion obey Fermi statistis). Similarily,

$$
CP \mid \pi^o \pi^o > = + \mid \pi^o \pi^o >. \tag{3}
$$

The three-pions states resulting from decay of neutral kaon are $\pi^+ \pi^- \pi^o$ and $3\pi^o$. Calling the relative angular momentum of π^+ and π^-, ℓ, and the angular momentum of π^o with respect to the center of mass of the two charged pion system L, the combined angular momentum should be zero (since kaon has zero spin). By conservation of angular momentum, $L = \ell$. The CP parity of the 3π system depends on the relative angular momentum of the $\pi^+\pi^-$ and π^o system. Explicitly,

$$
\begin{aligned}
CP \mid \pi^+ \pi^- \pi^o > &= \xi^{CP}_{\pi^+\pi^-} \xi^{CP}_{\pi^o} (-1)^L \mid \pi^+ \pi^- \pi^o > \\
&= (-1)^{L+1} \mid \pi^+ \pi^- \pi^o >.
\end{aligned} \tag{4}
$$

In $3\pi^0$ system L must be even, since two identical bosons must be symmetrical. Hence

$$CP \mid 3\pi^0 > \; = \; - \mid 3\pi^0 >. \qquad (5)$$

The CP parity of the short and long lived kaons is thus concluded to be even and odd respectively.

In attempt to reconcile CP invariance for all interactions, immediately after Christenson's experiment, Bell & Perring, and Bernstein, Cabibbo & Lee[5] independently postulated the existence of a new long range, extremely weak field, interacting between the kaon and our galaxy, producing a potential energy which is equal in magnitude but opposite in sign for the neutral kaon and its anti-particle. This interaction could generate the observed two pion decay of K_L without violating CP invariance, and implies the existence of a new particle with arbitrary spin J coupled to hypercharge (Y) or to Y plus some linear combination of electric charge (Q) and baryon number (N). The effect of this perturbing potential is to make the two eigenstates of kaon no longer exactly symmetric and antisymmetric under CP operation. Then the rate of decay (1) is predicted to be proportional to the $2J$ power of the kaon energy.

Levy & Nauenberg[6] postulated the existence of a vector particle S with mass less than the mass difference

of K_L and K_S and with intrinsic odd CP parity. The K_L can first decay into K_S and the vector particle S, the K_S then decay into two pion which is CP conserving.

Noticing that if the two-charged-pion events from a coherent beam of K_S and K_L mesons are truly due to the decays of K_S and K_L mesons and also the products are $\pi^+ \pi^-$ pairs, not two new particles, physicists suggested the two charged pions should show interference phenomena. Both models above definitely predict the absence of interference in the final 2π decay product since new particles are introduced. Fitch et al, Alff-steinberger et al, Bolt-Bodenhausen et al, Firestone et al[7] observed pronounced constructive interference in the two-pion final state of kaon to two pions decay, thus conclusively excluding models of kaon to two pions decay which involve new particles or fields.

Various experiments on reaction (1) at CERN, Princeton and the Rutherford Laboratory[8] with kaon monenta ranging from 1.1 to 10.7 Gev/c show no dependence of the decay rate on kaon energy, and the agreement between the various independent experiments is excellent. This further ruled out the explanation of reaction (1) by a long-range field.

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The interference experiments clearly establish CP violation in the weak interaction. Various CP violation theories were proposed. Sachs[9] assuming strict conservation of CP symmetry in nonleptonic interactions, suggested that CP noninvariance manifests itself in large violation of the $\Delta S = \Delta Q$ rule for semileptonic decays of hadrons, where ΔS, ΔQ refers to the change in strangeness and electric charge of the hadrons. A maximal violation of CP symmetry in semileptonic decay occurs when the $\Delta S = -\Delta Q$ interaction is 90 degrees out of phase with the $\Delta S = \Delta Q$ interaction. In order to obtain the result of Christenson, the magnitude of the $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ transition should be of the same order. He further discusses tests for CP violation in leptonic decays and indicate the inconclusiveness of the previous proofs of the $\Delta S = \Delta Q$ rule[10] which leaned heavily on the presumption of CP invariance. Experiments on the $\Delta S = \Delta Q$ rule were either remeasured or reanalyzed[11]. Their results do not support the required large violation of $\Delta S = \Delta Q$ rule, but because of the limited precision in these measurements, one cannot exclude a very small breakdown of the $\Delta S = \Delta Q$ rule.

Sachs' model was criticized by Wolfenstein[12] on the ground that:

1) It is impossible to write the Hamiltonian in the form of a current interacting with itself.

2) The amplitude of $\Delta S = -\Delta Q$ decays be comparable to that of $\Delta S = \Delta Q$ is quite improbable.

3) Arguments with respect to the real parts of the self-energy of kaon arising from leptonic intermediate state are much larger than that from nonleptonic are very difficult to make since both involve divergent integrals.

He suggested the possible existence of a new superweak interaction, by adding a small $\Delta S = -\Delta Q$ strangeness-changing current to the usual leptonic, strangeness-conserving, and $\Delta S = \Delta Q$ strangeness-changing currents, while writing his weak interaction hamiltonian in standard current-current form. This superweak interaction violates CP invariance and allows $\Delta S = 2$ transition with coupling constant between 10^{-7} and 10^{-8} times the usual weak interaction coupling constant (G). Lee & Wolfenstein[13] studied the general implication of such a superweak interaction and predicted that the ratio of the decay amplitudes to two charged and neutral pions would be the same for the short and long lived kaon, i.e.,

$$| \eta_{+-} | = | \eta_{oo} |, \tag{6}$$

where

$$\eta_{+-} = \frac{A (K_L \rightarrow \pi^+\pi^-)}{A (K_S \rightarrow \pi^+\pi^-)} ,$$

$$\eta_{oo} = \frac{A (K_L \rightarrow \pi^o\pi^o)}{A (K_S \rightarrow \pi^o\pi^o)} .$$

The two neutral pion decay mode of K_L was observed by Gaillard et al and Cronin et al[14] and the measured η_{oo} is at least twice or larger than η_{+-} , hence strongly object to the prediction of the superweak model.

Lee & Wolfenstein[13] also investigated the possibility of explaining the CP noninvariant reaction (1) by introducing a new strangeness-conserving but C-noninvariant and T-noninvariant interaction H_F whose coupling strength F is about 10^3 times the usual weak interaction coupling constant. Then the CP violating weak process would occur through the second order process described by $H_G H_F$, which contains terms that violate strangeness conservation, P-invariance, C-invariance, CP-invariance and T-invariance, where H_G is the usual weak interaction hamiltonian. H_F may be a small integral part of strong interaction, or a second order effect of C-nonconserving electromagnetic interaction H_γ. The possibility of a large C-violation in electromagnetic interaction has been pointed out by Bernstein, Feinberg & Lee[15]. The existence of H_F and C-violation in electromagnetic interactions can be tested by observing the energy asymmetry between π^+ and π^- in the 3π decay modes of η° and ω° as well as the radiative decay $\eta^\circ \to \pi^+ + \pi^- + \gamma$.

Subsequent experiments[16] however do not give strong evidence for such an asymmetry and are consistent with charge

conjugation invariance in electromagnetic interactions.

A related suggestion made by Prentki & Veltman[17] that C, T noninvariant interaction is simply the usual semistrong interaction; i.e., the SU(3) symmetry violating but SU(2) conserving part of usual strong interaction. However their theory cannot reconcile a sizable violation of C, T invariance with the extreme smallness of the observed CP noninvariant $K_L \to 2\pi$ decay and the high accuracy of C invariance in $\bar{p}p$ annihilation.

Cabibbo[18] summarizes the ideas of CP violation in lagrangian current-current models in terms of first and second class currents. He started by the assumption that CP violation can be large in weak interaction and be restricted to the hadronic currents which are members of a single octet of hermitian currents J^i_λ , (i runs from one to eight and λ is the space time index), and have definite CP behaviour,

$$CP\, J^{i\pm}_\lambda\, (CP)^{-1} = \pm\, \eta(i)\, J^{i\pm}_\lambda, \tag{7}$$

for $\lambda = 1,2,3$ and a minus sign for $\lambda = 4$, also $\eta(i) = +1$ for $i = 2,3,5,6,8$ and $\eta(i) = -1$ for $i = 1,4,7$. J^{i+}_λ and J^{i-}_λ are respectively first and second class currents. CP violation arises from the interference among first and second class current. Several authors[19] make use of second class

currents to construct models which gives C violation and thence CP violation in weak interactions. The existence of second class current will give rise to an inequality of the decay rates of $\Sigma^+ \to \Lambda e^+ \nu$ and $\Sigma^- \to \Lambda e^- \bar{\nu}$ other than the inequality arising from the difference in the masses of Σ^+, Σ^-. Barash's[20] experiment however has clearly indicated the absence of a second class current. The same experiment confirmed strongly the $\Delta S = \Delta Q$ rule.

The various suggestions on CP violation summarized above have not provided a satisfactory explanation for the CP noninvariance reaction (1). There remains the possibility of describing formally the known CP-violations using phenomenological field theory and first-class currents which is explored in this thesis. The present value of η_{+-} and η_{oo} lead to the belief in the existence of $\Delta I = 3/2$ and possibly $\Delta I = 5/2$ transition in the kaon to two pion decay, where ΔI is the change in isospin value of the initial and final states.

The phenomenological lagrangian is constructed in chapter II. Relation between the phase angle β_o and β_2 appearing in the definition of the lagrangian is determined in chapter III. The decay amplitude of kaon to two pions and the final state strong interaction $\pi - \pi$ phase shift

are calculated in chapter IV. The various CP violation parameters are calculated and compared with experiments in chapter V. A brief discussion on the possibility of describing CP violation processes in the baryon is made in chapter VI. The way to accommodiate some recent experimental reports which seems to disagree with our predictions, is also discussed.

CHAPTER II CONSTRUCTION OF A LAGRANGIAN MODEL

The chiral dynamics introduced by Schwinger[21] and others[22], using phenomenological fields and lagrangians, enables one to construct currents in terms of renormalized particle fields. The strong lagrangian L_S for interacting isodoublet nucleons and isotriplet pions, is invariant under the isospin rotation;

$$\begin{aligned} \phi &\to \phi - \delta\omega \wedge \phi, \\ \psi &\to (1 + \tfrac{i}{2}\tau \cdot \delta\omega)\psi, \end{aligned} \tag{8}$$

and apart from the pion mass term, invariant under the infinitesimal gauge transformation;

$$\begin{aligned} \phi &\to \phi + \delta\zeta + a^2\{2\phi\delta\zeta\cdot\phi + \delta\zeta\phi^2\}, \\ \psi &\to (1 + ia^2\tau\cdot\phi\wedge\delta\zeta)\psi, \end{aligned} \tag{9}$$

where $a^2 = (f_0/m_\pi)^2$ is the S-wave pion-nucleon interaction coupling constant. $\delta\omega$ and $\delta\zeta$ are infinitesimal group parameters. ϕ and ψ denotes the pion and nucleon field. ϕ, ψ, $\delta\omega$ and $\delta\zeta$ are isovectors. $\cdot$ and $\wedge$ denotes scalar and vector product in isospin space.

The combined transformation (8) and (9) on pion and nucleon fields forms two non-linear realizations of the same four-dimensional Euclidean rotation group O(4) which decomposes exactly into left and right handed SU(2) subgroups. (Appendix A)

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The strangeness-conserving hadronic vector and axial vector current, V_μ and A_μ, which participate in weak and electromagnetic processes are defined by the well known procedure of varying the lagrangian with the group parameters $\delta\omega$ and $\delta\zeta$ generalized to space-time functions. i.e.

$$\delta L_S = -V_\mu \partial^\mu \delta\omega - A_\mu \partial^\mu \delta\zeta + (\text{mass term}). \quad (10)$$

Strangeness-changing hadronic currents can be obtained by extending the group to include strange particles. One way of extending the group, studied by Zumino[22], is to consider only the isodoublet kaon and isotriplet pion system. The role of nucleon in the pion-nucleon system, is replaced by kaon in this system, and the kaon obeys the same transformation rule as the nucleon. The isodoublet strangeness-changing current, however, is not completely defined by the chiral $SU(2) \times SU(2)$ group, since the group does not contain strangeness-changing operators. Thus the isodoublet current contains one arbitrary constant, not an overall scale factor, of order unity.

Another way to include strange particle in chiral dynamic, is to start with the nonet of pseudoscalar mesons, and study the chiral $U(3)xU(3)$ group[23]. The strong interaction lagrangian of mesons which apart from the mass term is invariant under the chiral $U(3)xU(3)$ group, is defined as;

(11) $$L(\text{mesons}) = -\frac{1}{8f^2}\text{Trace}(\ \partial_\mu M^\dagger \partial^\mu M\ ) + \text{mass term},$$

where M and its hermitian conjugate $M^\dagger$, are the coupling matrix expandable as a series in Φ , the 3x3 hermitian pseudoscaler meson matrix. The strangeness-conserving and strangeness-changing hadronic currents can then be defined similarly to (10). Since the strange and non-strange currents are defined on the same footing, they are unambiguous. However, there is a second plausible choice for the currents, namely,

$$\sum_{k=1}^{9} (\ V_k^\mu + A_k^\mu\ )F_k \ = \ \partial^\mu M, \tag{12}$$

where F_k are the SU(3) operators, F_9 is the unit operator.

The ambiguity of choice here gives again an ambiguity in the isodoublet strange current of the same order as in Zumino's model. However, the ambiguity in the definition of strangeness-changing currents affects the calculated transition amplitude of kaon to two pion, by the additive factor of order $m_\pi^2/m_K^2 \simeq .06$ which is neglected in the subsequent calculations. For more complex processes, this ambiguity should be resolved.

Thus, in spite of the ambiguity, we can safely discuss the kaon to two pion process using these currents.

All currents obtained from chiral transformation are automatically first class with CP symmetry of the $V_\mu \pm A_\mu$ currents, (Appendix B),

$$J_\mu(\Delta I_3, \Delta Y)^{CP} = -e^{i\alpha\Delta Y}\varepsilon_\mu J_\mu(-\Delta I_3, -\Delta Y) \qquad (13)$$

where α is an arbitrary CP phase angle and $\varepsilon_{1,2,3} = -1$, $\varepsilon_4 = +1$. Second class currents, which have the opposite CP symmetry do not result from the chiral transformation.

CP conserving nonleptonic weak decays have been described by Nieh[24] using a lagrangian which is bilinear in the chiral vector currents. A CP noninvariant weak lagrangian can be constructed using the same set of currents.

The proposed weak interaction lagrangian is;

$$\begin{aligned} L_W(K\to 2\pi) = {} & G\{J_\mu(\pi^+)\cos\theta + e^{i\gamma}J_\mu(K^+)\sin\theta\}j_\mu(\ell\nu) \\ & + G_{1/2}e^{i\beta_0}\{J_\mu(K^+)J_\mu(\pi^-) - \tfrac{1}{\sqrt{2}}J_\mu(K^0)J_\mu(\pi^0)\} \\ & + G_{3/2}e^{i\beta_2}\{J_\mu(K^+)J_\mu(\pi^-) + \sqrt{2}J_\mu(K^0)J_\mu(\pi^0)\} \\ & + \text{adjoint}. \end{aligned} \qquad (14)$$

The hadronic currents J_μ are labelled by particles which carry the relevant quantum numbers. Thus $J_\mu(K^+)$ stands for $J_\mu(\Delta I_3 = \tfrac{1}{2}, \Delta Y = 1)$. j_μ is the leptonic current.

The leptonic current is coupled to the Cabibbo type[25] of octet current with a phase γ assigned to the strangeness

changing one. The phase factor γ does not give rise to any observable CP violation since the semileptonic and nonleptonic decays cannot interfere. It may be chosen to be zero.

A bilinear form in chiral current can give only $\Delta I = 1/2$ and $\Delta I = 3/2$ but not $\Delta I = 5/2$ transitions, since the maximum I possible is $3/2$ for the product of two octet currents. We construct the nonleptonic lagrangian by decomposing the product of currents into $\Delta I = 1/2$ and $\Delta I = 3/2$ parts with the appropiate Clebsch-Gorden coefficient and associating an independent phase factor with each part of the lagrangian.

The currents used in our model written explicitly in terms of the pion field π, and kaon field K, are;

For strangeness-conserving currents,

$$V_\mu(\pi^\pm) = \pm \pi^0 i\overleftrightarrow{\partial}_\mu \pi^\pm,$$

$$V_\mu(\pi^0) = \pi^+ i\overleftrightarrow{\partial}_\mu \pi^-,$$

$$A_\mu(\pi^\pm) = f_\pi \partial_\mu \pi^\pm + (4f_\pi)^{-2}\{2(\pi\cdot\partial_\mu\pi)\pi^\pm - 3\pi^2\partial_\mu\pi^\pm\},$$

$$A_\mu(\pi^0) = f_\pi \partial_\mu \pi^0 + (4f_\pi)^{-2}\{2(\pi\cdot\partial_\mu\pi)\pi^0 - 3\pi^2\partial_\mu\pi^0\},$$

where $$\pi\cdot\pi = \pi^+\pi^- + \pi^-\pi^+ + \pi^0\pi^0,$$

and for strangeness-changing currents,

$$V_\mu(K^+) = \frac{i}{2}\Big(\frac{f_K}{f_\pi}\Big)\{\partial_\mu(\pi^0 K^+ + \sqrt{2}\pi^+ K^0) - g(\partial_\mu \pi^0 K^+ + \sqrt{2}\partial_\mu \pi^+ K^0)\}$$
$$V_\mu(K^0) = \frac{i}{2}\Big(\frac{f_K}{f_\pi}\Big)\{\partial_\mu(\sqrt{2}\pi^- K^+ - \pi^0 K^0) - g(\sqrt{2}\partial_\mu \pi^- K^+ - \partial_\mu \pi^0 K^0)\} \quad (17)$$

$$A_\mu(K^+) = f_K \partial_\mu K^+ - (f_K/8f_\pi^2)\partial_\mu(\pi^2 K^+) + \quad \cdots$$
$$A_\mu(K^0) = f_K \partial_\mu K^0 - (f_K/8f_\pi^2)\partial_\mu(\pi^2 K^0) + \quad \cdots \quad (18)$$

where f_π and f_K are the pion and kaon decay constant, g is the undetermined constant discussed earlier.

A similar strangeness changing lagrangian as (14) would arise by extending the well known Cabibbo theory[25], in which currents containing a $\Delta S = 1$ component are obtained by rotation of the isospin triplet ($\Delta S = 0$) of currents over an angle around the seventh axis in SU(3) space. This rotation leads to a lagrangian describing CP conserving processes. The generalization is to rotate the currents around both the sixth and seventh axis through an angle θ, and take a linear combinations of the rotated currents to construct the lagrangian which violates CP symmetry. The weak lagrangian is written in the form;

$$L_W = G_8(\cos\beta' L_8^7 + \sin\beta' L_8^6)$$
$$+ G_{27}(\cos\beta L_{27}^7 + \sin\beta L_{27}^6), \quad (19)$$

where
$$L_j^i = e^{2i\theta F^i} L_j e^{-2i\theta F^i},$$

with i = 6 and 7 designates the axis of rotation, j = 8 or

[illegible]

27 designates the SU(3) multiplet of which the component L_j is a member. L_8 is the untary singlet combination of octet currents and

$$L_{27} = L_{27,0} + aL_{27,1} + bL_{27,2} ,$$

a and b are free parameters, while $L_{27,0}$, $L_{27,1}$ and $L_{27,2}$ are combinations of octet currents which induce $(\Delta I=0, \Delta I_3=0)$ $(\Delta I=1, \Delta I_3=0)$ and $(\Delta I=2, \Delta I_3=0)$ respectively before rotation through the Cabibbo angle. The rotated lagrangian contains a $\Delta S = 2$ transition which is strongly forbidden experimentilly. When the $\Delta S = 2$ transition is eliminated, the parameters a and b become linearly dependent. Then the strangeness-changing lagrangian, in terms of the rotated currents, becomes,

$$\begin{aligned} L_w \sim\ & G\mathcal{g}e^{-i\beta'}\Big(J_\mu(K^+)J_\mu(\pi^-) - \tfrac{1}{\sqrt{2}}J_\mu(K^0)J_\mu(\pi^0) \\ & \quad - \tfrac{1}{\sqrt{6}}J_\mu(K^0)J_\mu(\eta^0)\Big) \\ & + G\mathcal{g}_{27}e^{-i\beta}\Big(J_\mu(K^+)J_\mu(\pi^-) - \tfrac{1}{\sqrt{2}}J_\mu(K^0)J_\mu(\pi^0) \\ & \quad + \tfrac{3\sqrt{3}}{2}J_\mu(K^0)J_\mu(\eta^0) \\ & \quad + 5\{J_\mu(K^+)J_\mu(\pi^-) + \sqrt{2}J_\mu(K^0)J_\mu(\pi^0)\}\Big) \\ & + \text{adjoint.} \end{aligned} \tag{20}$$

However if one uses only phenomenological currents, the term $J_\mu(K^0)J_\mu(\eta^0)$ does not contribute to either $K\to 2\pi$

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or $K \to 3\pi$ process (Appendix C). Thus the above lagrangian (20) reduces to the nonleptonic part of (14).

The proposed lagrangian can also be obtained by supposing that each quartet of fields having the quantum numbers of $(K, \bar{K})$ behaves like an isotriplet plus an iso-singlet with respect to weak interactions, but with the triplet-singlet decomposition dependent on the particular interaction. The strange currents $J_\mu(K)$ will then have similar "schizon" properties.[26] Thus one may suppose that $J_\mu(K)$ decomposes into the triplet,

$$\left(J_\mu(K^+)e^{i\xi}\ ,\ \frac{1}{\sqrt{2}}\{J_\mu(K^0)e^{i\xi} + J_\mu(\bar{K}^0)e^{-i\xi}\},\ J_\mu(K^-)e^{-i\xi}\right)$$

and singlet,

$$J_\mu(K_2^0) = \frac{1}{\sqrt{2}}\{J_\mu(K^0)e^{i\xi} - J_\mu(\bar{K}^0)e^{-i\xi}\}$$

where ξ is γ, β_0 or β_2.

Then the Cabibbo current in Eq. (14) coupled to leptons is the charged component of a composite isotriplet while the 1/2 part of L_W is a "I" = 0 and 3/2 part a "I" = 2 spurion.

Both arguments should be regarded as heuristic only. The validity of the model is determined by its observable predictions independently of the argument used to find it.

We proceed with the lagrangian (14) in the following discussions.

CHAPTER III DETERMINATION OF THE RELATION BETWEEN β_0 AND β_2 .

Terms in $L_W(K\to 2\pi)$ which contribute to $K^0\to 2\pi$ decays to the lowest order in f may be isolated by using the explicit expression for each current in the phenomenological meson fields.

The product of currents $J_\mu(K^+)J_\mu(\pi^-)$ and $J_\mu(K^0)J_\mu(\pi^0)$ written out explicitly are;

$$\begin{aligned} J_\mu(K^+)J_\mu(\pi^-) = (i/2)f_K\{&(1-g)(\partial_\mu\pi^0)(\partial_\mu\pi^-)K^+ + (\pi^0\partial_\mu\pi^-)\partial_\mu K^+ \\ &+ \sqrt{2}(1-g)(\partial_\mu\pi^+)(\partial_\mu\pi^-)K^0 \\ &+ \sqrt{2}(\pi^+\partial_\mu\pi^-)\partial_\mu K^0\} \\ &- f_K(\pi^0 i\overleftrightarrow{\partial}_\mu\pi^-)\partial_\mu K^+, \end{aligned} \tag{21}$$

and

$$\begin{aligned} J_\mu(K^0)J_\mu(\pi^0) = (i/2)f_K\{&\sqrt{2}(1-g)(\partial_\mu\pi^-)(\partial_\mu\pi^0)K^+ \\ &+\sqrt{2}\pi^-(\partial_\mu\pi^0)\partial_\mu K^+ - (\pi^0\partial_\mu\pi^0)\partial_\mu K^0 \\ &- (1-g)(\partial_\mu\pi^0)(\partial_\mu\pi^0)K^0\} \\ &+ f_K(\pi^+ i\overleftrightarrow{\partial}_\mu\pi^-)\partial_\mu K . \end{aligned} \tag{22}$$

The current $\pi^+ i\overleftrightarrow{\partial}_\mu\pi^-$ is a conserved vector current, hence the term $(\pi^+ i\overleftrightarrow{\partial}_\mu\pi^-)\partial_\mu K^0$ gives no contribution to the

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decay amplitude. Similarly the contribution to the decay amplitude from the term $\pi^0 i\overleftrightarrow{\partial}_\mu \pi^- \partial_\mu K^+$ is zero, when the small difference between the neutral and charged pion mass is neglected.

The weak lagrangian responsible for $K^\pm \to 2\pi$ and neutral kaon to 2π can be written as;

$$L_w(K^\pm \to 2\pi) = iG_{3/2} e^{i\beta_2} (3/2) f_K \Big((1-g)(\partial_\mu \pi^0)(\partial_\mu \pi^-) K^+ + (\pi^0 \partial_\mu \pi^-) \partial_\mu K^+ \Big) + \text{adjoint}, \tag{23}$$

and

$$\begin{aligned} L_w(K^0 \to 2\pi) = {} & (i/\sqrt{2}) f_K G_{1/2} e^{i\beta_0} \times \\ & \Big((1-g)\{(\partial_\mu \pi^+)(\partial_\mu \pi^-) + \tfrac{1}{2}(\partial_\mu \pi^0)(\partial_\mu \pi^0)\} K^0 \\ & + \{(\pi^+ \partial_\mu \pi^-) + \tfrac{1}{2}(\pi^0 \partial_\mu \pi^0)\} \partial_\mu K^0 \Big) \\ & + (i/\sqrt{2}) f_K G_{3/2} e^{i\beta_2} \times \\ & \Big((1-g)\{(\partial_\mu \pi^+)(\partial_\mu \pi^-) - (\partial_\mu \pi^0)(\partial_\mu \pi^0)\} K^0 \\ & + \{(\pi^+ \partial_\mu \pi^-) - (\pi^0 \partial_\mu \pi^0)\} \partial_\mu K^0 \Big) \\ & + \text{adjoint}, \end{aligned} \tag{24}$$

where g is the undetermined constant.

We write,

$$\Psi_{0\,\mu}^{\ \mu} = (2/\sqrt{3})\{(\partial_\mu\pi^+)(\partial^\mu\pi^-) + \tfrac{1}{2}(\partial_\mu\pi^0)(\partial^\mu\pi^0)\} = \overline{\Psi}_{0\,\mu}^{\ \mu} ,$$

$$\Psi_{2\,\mu}^{\ \mu} = (2/\sqrt{6})\{(\partial_\mu\pi^+)(\partial^\mu\pi^-) - (\partial_\mu\pi^0)(\partial^\mu\pi^0)\} = \overline{\Psi}_{2\,\mu}^{\ \mu} ,$$

$$\Psi_{0\,\mu} = (2/\sqrt{3})\{\pi^+\partial_\mu\pi^- + \tfrac{1}{2}\pi^0\partial_\mu\pi^0\},$$

$$\Psi_{2\,\mu} = (2/\sqrt{6})\{\pi^+\partial_\mu\pi^- - \pi^0\partial_\mu\pi^0\}, \tag{25}$$

$$\overline{\Psi}_{0\,\mu} = (2/\sqrt{3})\{\pi^-\partial_\mu\pi^+ + \tfrac{1}{2}\pi^0\partial_\mu\pi^0\},$$

$$\overline{\Psi}_{2\,\mu} = (2/\sqrt{6})\{\pi^-\partial_\mu\pi^+ - \pi^0\partial_\mu\pi^0\}.$$

Then we define,

$$2\Psi^+_{0\,\mu} = (2/\sqrt{3})\{\pi^+\partial_\mu\pi^- + \pi^-\partial_\mu\pi^+ + \pi^0\partial_\mu\pi^0\} = \Psi_{0\,\mu} + \overline{\Psi}_{0\,\mu},$$

$$2\Psi^+_{2\,\mu} = (2/\sqrt{6})\{\pi^+\partial_\mu\pi^- + \pi^-\partial_\mu\pi^+ - 2\pi^0\partial_\mu\pi^0\} = \Psi_{2\,\mu} + \overline{\Psi}_{2\,\mu},$$

$$2\Psi^-_{0\,\mu} = (2/\sqrt{3})(\pi^+\overleftrightarrow{\partial}_\mu\pi^-) = \Psi_{0\,\mu} - \overline{\Psi}_{0\,\mu}, \tag{26}$$

$$2\Psi^-_{2\,\mu} = (2/\sqrt{6})(\pi^+\overleftrightarrow{\partial}_\mu\pi^-) = \Psi_{2\,\mu} - \overline{\Psi}_{2\,\mu}.$$

$\Psi^-_{0\,\mu}$ and $\Psi^-_{2\,\mu}$ again are conserved vector currents which give no contribution to the decay amplitude. The bar over the field represents its hermitian conjugate. $\Psi_{0\,\mu}^{\ \mu}$, $\Psi^+_{0\,\mu}$ and $\Psi_{2\,\mu}^{\ \mu}$ $\Psi^+_{2\,\mu}$ are the pion field combinations which act on vaccum to create the normalized 2π state with isospin equal to zero and two.

The particle states $|K_S\rangle$ and $|K_L\rangle$, combinations

of the eigenstates $|K^0\rangle$, $|\bar{K}^0\rangle$ of the strong interaction, are defined by the two conditions:

1) They are orthogonal.
2) They decay into mutually orthogonal two-pion states.

The second condition is needed to elimenate the second order weak process $|K_S\rangle \to |2\pi\rangle \to |K_L\rangle$, so that the orthogonality is essentially conserved by the CP violating weak interaction. Some small mixing may still remain due to 3π and leptonic intermediate states, but this can be safely neglected.

We define

$$|K_S\rangle = (1/\sqrt{2})\{e^{i\alpha}|K^0\rangle + e^{-i\alpha}|\bar{K}^0\rangle\},$$
$$|K_L\rangle = (1/\sqrt{2})\{e^{i\alpha}|K^0\rangle - e^{-i\alpha}|\bar{K}^0\rangle\}. \tag{27}$$

Adopting the Wigner phase convention in which the fields, K^0 and $\bar{K}^0$, when acting on the corresponding particle state, gives positive and negative vaccum state, $|0\rangle$, respectively. Then the fields corresponding to the particle states $|K_S\rangle$ and $|K_L\rangle$ are;

$$K_S = (i/\sqrt{2})\{e^{-i\alpha}K^0 - e^{i\alpha}\bar{K}^0\},$$
$$K_L = (1/\sqrt{2})\{e^{-i\alpha}K^0 + e^{i\alpha}\bar{K}^0\}. \tag{28}$$

The incorporation of an i in the definition of K_S

makes K_S a hermitian field.

Then the action of the fields K_S and K_L on the corresponding particle states are;

$$K_S\ |K_S\rangle = i|0\rangle,$$

$$K_L\ |K_L\rangle = +|0\rangle.$$

The K^0 and $\overline{K}^0$ fields in term of fields K_S and K_L are;

$$\begin{aligned} K^0 &= -(i/\sqrt{2})e^{i\alpha}\{K_S + iK_L\}, \\ \overline{K}^0 &= +(i/\sqrt{2})e^{-i\alpha}\{K_S - iK_L\}. \end{aligned} \tag{29}$$

With equations (25), (26) and (29), the lagrangian (24) can be transformed into;

$$L_w(K^0\to 2\pi) = -(\sqrt{3}/2)f_K G_{1/2}\{\ (1-g)(\ \Psi_{S\mu}^{\ \mu},\ \ \Psi_{L\mu}^{\ \mu})\begin{pmatrix} K_S \\ K_L \end{pmatrix} + (\ \Psi_{S\mu}\ ,\ \Psi_{L\mu}\ )\partial^\mu\begin{pmatrix} K_S \\ K_L \end{pmatrix}\}\ , \tag{30}$$

where,

$$(\ \Psi_{S\mu}^{\ \mu}\ ,\ \Psi_{L\mu}^{\ \mu}\ ) = (\ \Psi_{0\ \mu}^{\ \mu}\ ,\ \Psi_{2\ \mu}^{\ \mu}\ )(A)$$

$$(\ \Psi_{S\mu}\ ,\ \Psi_{L\mu}\ ) = (\ \Psi_{0\ \mu}^{\ +}\ ,\ \Psi_{2\ \mu}^{\ +}\ )(A)$$

$$(A) = \begin{pmatrix} \cos(\beta_0+\alpha) & -r\sqrt{2}\sin(\beta_0+\alpha) \\ \cos(\beta_2+\alpha) & -r\sqrt{2}\sin(\beta_2+\alpha) \end{pmatrix}$$

and $r = G_{3/2}/G_{1/2}$.

Thus, a single transformation matrix (A), transforms both terms of L_W into expressions in the observable fields.

Since the states Ψ_0 and Ψ_2 are orthogonal, the required orthogonality of the states $|\Psi_S>$, $|\Psi_L>$ demands relation between the angles, $(\beta_0+\alpha)$ and $(\beta_2+\alpha)$, as;

$$\sin 2(\beta_0+\alpha) = -2r^2 \sin 2(\beta_2+\alpha) \tag{30}$$

The mixing angle α may be chosen zero, without observable consequences, so that K_S and K_L are CPT self-conjugate as well as hermitian fields. The possibility of this choice was pointed out earlier by Mathur[27].

A similar result, Eq. (30), was obtained by using the formulism of Wu and Yang[28], assuming that the phases of the off diagonal element of the mass and decay matrix of kaon are equal (Appendix D). But, since this argument involves a divergent self-mass integral, which is unacceptable in a phenomenological theory, the earlier argument is perhaps preferable.

The chosen CPT phase convention effectively assigns most of the CP violation to the $\Delta I = 3/2$ part of the lagrangian; but the assign-ment is a convention, since the CP violation arises from the interference

between the two decay channels. If only the 1/2 part of L_W were present, for example, the phase factor $e^{i\beta_0}$ could be eliminated by redefining the particle states $|K^0\rangle$ and $|\overline{K}^0\rangle$ as $|K^0\rangle e^{i\beta_0}$ and $|\overline{K}^0\rangle e^{-i\beta_0}$ respectively. Thus the CP violation occurs only because both 3/2 and 1/2 parts are present, so that the relative phase cannot be transformed to zero. In a contrary interpretation the assignment is regarded as physically significant rather than merely conventional[27].

CHAPTER IV CALCULATION OF DECAY AMPLITUDE AND FINAL STATE $\pi - \pi$ PHASE SHIFT

The final state $\pi - \pi$ strong interaction phase shift in small momentum approximation could be calculated by using the results of chiral dynamics. The $\pi - \pi$ scattering amplitudes given by Schwinger[21] are;

$$\begin{pmatrix} a_0 \\ a_2 \end{pmatrix} = \frac{f_0^2}{4\pi m_\pi} \begin{pmatrix} 6 - \frac{5}{2}(1+b) \\ -(1+b) \end{pmatrix} .$$

The subscript on the amplitudes refer to the iso-spin value $I = 0$ or 2. Under the special PCAC framework where b is taken to be 1/2, this becomes,

$$\begin{pmatrix} a_0 \\ a_2 \end{pmatrix} = \frac{f_0^2}{4\pi m_\pi} \begin{pmatrix} 9/4 \\ -3/2 \end{pmatrix} ,$$

with

$$(f_0^2/4\pi) = 0.050.$$

Then the phase shifts in the $I = 0, 2$ states is given by the well known relation,

$$a_I = (1/2ik)(e^{2i\delta_I} - 1) = (e^{i\delta_I}/k)\sin\delta_I ,$$

or

$$\sin\delta_I = k|a_I| .$$

In the rest frame of decaying kaon, the energy of

[illegible]

each pion is equal to one half the mass of kaon, i.e., 249 Mev. Hence, the momentum of the pion is;

$$\hbar k = p_\pi = \sqrt{E_\pi^2 - m_\pi^2} \simeq 207 \text{ Mev/c},$$

and,

$$\frac{\hbar k}{m_\pi} \simeq \frac{207}{139} \simeq 1.49.$$

Then

$$\delta_0 \simeq \sin\delta_0 \simeq .169 \text{ rad.}$$

$$\delta_2 \simeq \sin\delta_2 \simeq .112 \text{ rad.}$$

or

$$\delta = \delta_2 - \delta_0 \simeq -.281 \text{ rad.} \simeq -17 \text{ degrees.}$$

Since the momentum transfer is $1/4 m_\rho c$ this approximation should be valid. This value is compatible with $10^\circ \leq \delta_0 - \delta_2 \leq 60^\circ$ given in Glashow's paper[29].

The decay amplitudes for kaon to two pions are calculated using L_W in first order perturbation approximation. This is sufficient for the $K \rightarrow 2\pi$ process. For more complex processes, such as the $K \rightarrow 3\pi$ decays, one must include matrix elements from the combination $L_{Strong} L_W$ as calculated by Nieh[24].

The decay amplitude of,

$$K^+(p) \rightarrow \pi^+(k^+) + \pi^0(k^0),$$

where p, k^+ and k^0 denote the four momentum of the particle K^+, π^+ and π^0 respectively, is calculated in the frame where the decaying kaon is at rest. Using the lagrangian (23), and assuming that the kaon and pion are on the mass shell, we have;

$$A(K^+\rightarrow\pi^+\pi^0) = i(3/2)f_K G_{3/2} e^{i\beta_2} \times$$

$$\{(1-g)\langle\pi^+\pi^0|\partial_\mu\pi^0|\pi^+\rangle\langle\pi^+|(\partial_\mu\pi^-)K^+|K^+\rangle$$

$$+ \langle\pi^+\pi^0|\pi^0\partial_\mu\pi^-|0\rangle\langle 0|\partial_\mu K^+|K^+\rangle\}$$

$$= i(3/2)f_K G_{3/2} e^{i\beta_2} \times$$

$$\{(1-g)(-ik^0_\mu)(-ik^+_\mu) + (-ik^+_\mu)(ip_\mu)\}.$$

As $k^0_\mu k^+_\mu = E^0E^+ - \underset{\sim}{k}^+\cdot\underset{\sim}{k}$, where $E = k_4$ and $\underset{\sim}{k}$ is the three momentum vector, with $\underset{\sim}{k}^+ = -\underset{\sim}{k}^0$, we have;

$$k^0_\mu k^+_\mu = 2(E^+)^2 - m_\pi^2 = 2(E^0)^2 - m_\pi^2 = \tfrac{1}{2}M_K^2 - m_\pi^2.$$

The 'on mass shell' condition has been used in the above equation.

Similarily, we have,

$$k^+_\mu p_\mu = E^+M_K = \frac{1}{2} M_K^2 .$$

Thus neglecting term of order m_π^2/M_K^2 we have,

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$$A\{\ |K^+\rangle \rightarrow (1/\sqrt{2})|\pi^+\pi^0+\pi^0\pi^+\rangle\ \}$$

$$= 3\sqrt{2}(iD)G_{3/2}e^{i\beta_2},$$

where $D = (gf_K M_K^2/4)$ is a constant.

Similarily the matrix elements of Ψ_0 and Ψ_2 are;

$$\langle 2\pi_{I=0}|\ \Psi_{0\,\mu}^{\ \mu}\ |0\rangle$$

$$= (2/\sqrt{3})\langle 2\pi_{I=0}|\partial_\mu\pi^+\partial_\mu\pi^- + \tfrac{1}{2}\partial_\mu\pi^0\partial_\mu\pi^0|0\rangle \quad = \quad M_K^2 e^{i\delta_0},$$

$$\langle 2\pi_{I=2}|\ \Psi_{2\,\mu}^{\ \mu}\ |0\rangle$$

$$= (2/\sqrt{6})\langle 2\pi_{I=2}|\partial_\mu\pi^+\partial_\mu\pi^- - \partial_\mu\pi^0\partial_\mu\pi^0|0\rangle \quad = \quad M_K^2 e^{i\delta_2},$$

and,

$$\langle 2\pi_{I=0}|\Psi_{0\,\mu}^{+}|0\rangle\langle 0|\partial_\mu(K_S,K_L)|(K_S,K_L)\rangle$$

$$= (1/\sqrt{3})\langle 2\pi_{I=0}|\pi^+\partial_\mu\pi^- + \pi^-\partial_\mu\pi^+ \div \pi^0\partial_\mu\pi^0|0\rangle \times$$
$$\langle 0|\partial_\mu(K_S,K_L)|(K_S,K_L)\rangle$$

$$= \quad M_K^2 e^{i\delta_0}(i,1),$$

$$\langle 2\pi_{I=2}|\Psi_{2\,\mu}^{+}|0\rangle\langle 0|\partial_\mu(K_S,K_L)|(K_S,K_L)\rangle$$

$$= (1/\sqrt{6})\langle 2\pi_{I=2}|\pi^+\partial_\mu\pi^- + \pi^-\partial_\mu\pi^+ - 2\pi^0\partial_\mu\pi^0|0\rangle \times$$
$$\langle 0|\partial_\mu(K_S,K_L)|(K_S,K_L)\rangle$$

$$= -M_K^2 e^{i\delta_2}(i,1).$$

The final state $\pi-\pi$ phase shift δ, has been added to the calculated amplitude.

Thus the amplitudes of K_S and K_L to two pion decay are as follows;

$$A(K_S \to 2\pi) = 2\sqrt{3}(iD)G_{1/2}\{e^{i\delta_0}\cos(\beta_0+\alpha) + \sqrt{2}re^{i\delta_2}\cos(\beta_2+\alpha)\},$$

and

$$A(K_L \to 2\pi) = -2\sqrt{3}(D)G_{1/2}\{e^{i\delta_0}\sin(\beta_0+\alpha) + \sqrt{2}re^{i\delta_2}\sin(\beta_2+\alpha)\}.$$

Using the relations,

$$|\pi^+\pi^-> = (1/\sqrt{3})(\sqrt{2}|2\pi_{I=0}> + |2\pi_{I=2}>),$$

and

$$|\pi^0\pi^0> = -(1/\sqrt{3})(|2\pi_{I=0}> - \sqrt{2}|2\pi_{I=2}>).$$

we have,

$$A(K_S \to \pi^+\pi^-) = 2\sqrt{2}(iD)G_{1/2}\{e^{i\delta_0}\cos(\beta_0+\alpha) + re^{i\delta_2}\cos(\beta_2+\alpha)\},$$

$$A(K_S \to \pi^0\pi^0) = -2(iD)G_{1/2}\{e^{i\delta_0}\cos(\beta_0+\alpha) - 2re^{i\delta_2}\cos(\beta_2+\alpha)\},$$

$$A(K_L \to \pi^+\pi^-) = -2\sqrt{2}(D)G_{1/2}\{e^{i\delta_0}\sin(\beta_0+\alpha) + re^{i\delta_2}\sin(\beta_2+\alpha)\},$$

$$A(K_L \to \pi^0\pi^0) = 2(D)G_{1/2}\{e^{i\delta_0}\sin(\beta_0+\alpha) - 2re^{i\delta_2}\sin(\beta_2+\alpha)\}.$$

The above relations for the decay amplitude can be simplified by using Eq. (30) and the fact that both angles β_0 and β_2 are small, also α can be chosen as zero, hence $\cos\beta_0$ and $\cos\beta_2$ can be taken as equal to one. After taking the common factor $2\sqrt{2}(iD)G_{1/2}e^{i\delta_0}$ out from the amplitudes, and making the above simplification, we have the calculated amplitudes in the following ratio;

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$$|A(K^+\to\pi^+\pi^0)| \sim (3/2)r$$

$$A(K_S\to\pi^+\pi^-) \sim (1 + re^{i\delta})$$

$$A(K_S\to\pi^0\pi^0) \sim -(1/\sqrt{2})(1 - 2re^{i\delta})$$

$$A(K_L\to\pi^+\pi^-) \sim r\sin\beta_2 e^{i\{\delta+(\pi/2)\}}(1 - 2re^{-i\delta})$$

$$A(K_L\to\pi^0\pi^0) \sim \sqrt{2}r\sin\beta_2 e^{i\{\delta+(\pi/2)\}}(1 + re^{-i\delta}).$$

CHAPTER V DETERMINATION OF r, β_2, η_{+-} and η_{00}

The constant $r = G_{3/2}/G_{1/2}$ can be determined by using the experimental value of the decay rates of $K^+ \to \pi^+\pi^0$ and $K_S \to 2\pi$ as input.

In term of the calculated decay amplitudes, we have the decay rates;

$$R(K^+ \to \pi^+\pi^0) = |3\sqrt{2}G_{1/2} r e^{i\beta_2} D|^2$$

$$= 18G_{1/2}^2 r^2 D^2,$$

and

$$R(K_S \to 2\pi) = |A(K_S \to \pi^+\pi^-)|^2 + |A(K_S \to \pi^0\pi^0)|^2$$

$$= 12D^2G_{1/2}^2\{\cos^2(\beta_0+\alpha) + 2r^2\cos^2(\beta_2+\alpha)\}$$

$$\simeq 12D^2G_{1/2}^2(\ 1 + 2r^2\ ).$$

Thus we have,

$$R(K^+ \to \pi^+\pi^0)\ /\ R(K_S \to 2\pi)$$

$$= 18G_{1/2}^2 r^2 D^2\ /\ 12D^2G_{1/2}^2(1+2r^2)$$

$$= 3r^2\ /\ 2(1+2r^2).$$

The experimental decay rates of $K^+ \to \pi^+\pi^0$ and $K_S \to 2\pi$ are 1.695×10^7 and 1.145×10^{10} sec^{-1} respectively[30]. Using these values, the constant r is determined to be 3.15×10^{-2}.

The CP violation parameters, η_{+-} and η_{00} written in terms of the calculated amplitudes are;

$$\eta_{+-} = A(K_L \to \pi^+\pi^-) / A(K_S \to \pi^+\pi^-)$$

$$= \frac{r\sin\beta_2 e^{i\{\delta+(\pi/2)\}}(1 - 2re^{-i\delta})}{1 + re^{i\delta}}$$

$$\simeq r\sin\beta_2 e^{i\{\delta+(\pi/2)\}}(1 - 3r\cos\delta + ir\sin\delta),$$

and

$$\eta_{00} = A(K_L \to \pi^0\pi^0) / A(K_S \to \pi^0\pi^0)$$

$$= \frac{-2r\sin\beta_2 e^{i\{\delta+(\pi/2)\}}(1 + re^{-i\delta})}{1 - 2re^{i\delta}}$$

$$\simeq 2r\sin\beta_2 e^{-i\{(\pi/2)-\delta\}}(1 + 3r\cos\delta + ir\sin\delta).$$

The phase angles of η_{+-} and η_{00} denoted by ϕ_{+-} and ϕ_{00} are;

$$\phi_{+-} = \delta + (\pi/2) + (r\sin\delta)/(1 - 3r\cos\delta),$$

$$\phi_{00} = -\{(\pi/2) - \delta - (r\sin\delta)/(1 + 3r\cos\delta)\}.$$

Using the values $r = 3.15\times10^{-2}$ and $\delta = 17^\circ$, we have,

$$\phi_{+-} \simeq 73^\circ \quad \text{and} \quad \phi_{00} \simeq -106^\circ.$$

The predicted value of ϕ_{+-} falls within one standard deviation of the world averaged value, while the experimental value of ϕ_{00} is yet uncertain.

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The ratio,

$$\frac{|\eta_{00}|}{|\eta_{+-}|} = \frac{2(1+3r\cos\delta)}{(1-3r\cos\delta)} \simeq 2.42$$

lies between the experimental values reported by Gaillard and Cronin[18].

The angle β_2 can also be determined by using $|\eta_{+-}| = 1.89\times10^{-3}$ as input. We have,

$$\beta_2 \simeq \sin\beta_2 = |\eta_{+-}|/r(1-3r\cos\delta)$$

$$= 0.067 \text{ rad.}$$

The branching ratio of $K_S\to\pi^0\pi^0$ is

$$R(K_S\to\pi^0\pi^0)/R(K_S\to2\pi)$$

$$= (1 - 4r\cos\delta + r^2)/3(1 + 2r^2)$$

$$\simeq 2.93.$$

This value falls within the two standard deviation limit of the world average.

Predicted values of branching ratio and CP violation parameters are compared with available data in the table.

Parameter	Calculated	Measured.
ϕ_{+-}	73°	$(64\pm11)^{\circ}$ (30)
ϕ_{00}	-106°	not available
$\lvert\eta_{00}\rvert/\lvert\eta_{+-}\rvert$	2.42	2.5±0.2 2.2±0.3 (14)
$\dfrac{R(K_S\to\pi^0\pi^0)}{R(K_S\to 2\pi)}$	0.293	0.316±0.01 (30)

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CHAPTER VI CONCLUSION AND DISCUSSION

The success of the above model in predicting the correct ratio of $|\eta_{+-}|/|\eta_{00}|$ seems to confirm the substantial contribution of the $\Delta I = 3/2$ part of the lagrangian to CP violation. The correct prediction of the phase angle ϕ_{+-} which is calculated using the strong interact-interaction phase shift δ in the 2π final states, supports the PCAC hypothesis, since the calculation of δ is based on this hypothesis.

The slight disagreement of the predicted branching ratio of K_S to 2π indicated that some $\Delta I = 5/2$ transition amplitude might be required. It has been estimated[31] that the $\Delta I = 5/2$ coupling constant $G_{5/2}$ is about 9% of $G_{3/2}$ when accepting the present world average value of the branching ratio and choosing the phase which minimizes the $G_{5/2}$.

This model predicts no CP violation amplitude in the neutral kaon to three pion decay, since the isospin of the three pion state with total charge zero, can only be one or three. Anyhow the CP violating process $K_S \to 3\pi^0$, and a large decay amplitude of $K_S \to \pi^+\pi^-\pi^0$ (larger than that allowed by the centrifugal barrier), if observed experimentelly, could be accommodated by including a $\Delta I = 5/2$

part into the Lagrangian.

Extending the mesonic currents to accommodate the baryonic currents enables one to study CP violation in baryon decays. The chiral currents will now be in the form;

$$J^k_\mu = J^k_\mu(\text{mesons}) + \overline{B}\gamma_\mu\{1 + i(G_A/G_V)_k\gamma^5\}F_k B,$$

where B is the octet of baryons and F_k is the SU(3) generator.

The weak Lagrangian then describe processes like,

$$|\Lambda^0\rangle \rightarrow |p\pi^-\rangle \quad , \quad |\Lambda^0\rangle \rightarrow |n\pi^0\rangle,$$

and $$|\Xi^-\rangle \rightarrow |\Lambda^0\pi^-\rangle \quad , \quad |\Xi^0\rangle \rightarrow |\Lambda^0\pi^0\rangle,$$

which would be CP violating, since the decay processes utilize both the $\Delta I = 1/2$ and $\Delta I = 3/2$ channels. As in the Λ^0 decay, the final states $|p\pi^-\rangle$ and $|n\pi^0\rangle$ can have isospin 1/2 and 3/2 while the isospin of Λ^0 is zero. In the Ξ decay, the state $|\Lambda\pi\rangle$ has I=1 while Ξ has I=1/2.

However the processes,

$$|\Sigma^\pm\rangle \rightarrow |n\pi^\pm\rangle \quad , \quad |\Sigma^+\rangle \rightarrow |p\pi^0\rangle,$$

are CP conserving since the decay to both I=1/2 and I=3/2 states utilizes only the I = 1/2 channel.

But due to the spin of the baryons, any T odd correlation is associated with P odd correlation.[32] So to observe CP violation, we must determine the degree of P

violation and also correct the correlation for strong phase shift. The predicted small CP violation is practically not measurable.

The term $J_\mu(K^0)J_\mu(\eta^0)$ which we have excluded in the kaon to two- and three-pion decay processes, could describe the weak interactions such as;

$$\eta + \pi \rightarrow K + \pi, \qquad \eta + N \rightarrow N + K,$$

which may be CP violating as well as P violating. The magnitude of the coupling constant is as yet undetermined.

Our current-current Lagrangian model in its present form does not agree with a few recent experimental reports. However, by making a slight modification, as described below, these disagreement could be reconciled.

The reported charge asymmetry in the decay of $K_L \rightarrow e^\pm + \pi^\mp + \nu$, contradicts our assumption on the orthogonality of the K_S and K_L states. Since, the semileptonic decays of K_L is CP conserving, and assuming the validity of $\Delta Q = \Delta S$ rule, the decay to $e^+\pi^-\nu$ and $e^-\pi^+\bar{\nu}$ come from K^0 and $\bar{K}^0$ respectively, in the K_L state, which should contribute equally in our model. This asymmetry would imply some mixing of our orthogonal K_S and K_L states. However, if this charge asymmetry is confirmed, the mixing in the states can be accommodated in our model by

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adding a $\Delta S = 2$ term, $e^{i\beta_3}J_\mu(K^0)J_\mu(K^0)$ + adjoint, to our Lagrangian. Then, the Lee-Wolftenstein's[13] superweak interaction is included in our model. The particle states $|K_S>$ and $|K_L>$ would now be.

$$|K_S> \;=\; N\{(1+\varepsilon)K^0 + (1-\varepsilon)\bar{K}^0\},$$

$$|K_L> \;=\; N\{(1+\varepsilon)K^0 - (1-\varepsilon)\bar{K}^0\},$$

with $|1+\varepsilon| \neq 1$, and N is a normalization constant, insted of Eq. (27). This will result in a reduction of the calculated phase ϕ_{+-} of η_{+-}, which could now lie between the limits,

$$42^\circ < \phi_{+-} < 73^\circ,$$

where the value $\phi_{+-} = 42^\circ$ is calculated on the assumption of pure superweak interaction. The ratio $|\eta_{00}|/|\eta_{+-}|$ will also be reduced. Recent reported experimental values of $\phi_{+-} = 53^\circ$ and the low value of η_{00}[34,35], suggesting that the CP violating part of the Lagrangian is a mixture of $\Delta S = 1$ and $\Delta S = 2$ transition.

Reliable values of the mixing parameter, Reε, Imε, as well as η_{+-} will be needed to verify the PCAC hypothesis. Further development of the chiral dynamics would remove the ambiguity in the definition of the strangeness-changing currents. Also more accurate calculations and measurements of the 3π branching ratio, as well as the observation of

other CP violation processes would provide a further test of the current-current Lagrangian model.

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APPENDIX A PROOF OF $O(4) = SU(2) \times SU(2)$

According to Schwinger[21], the isospin rotation is,

$$\begin{aligned} \psi &\rightarrow \left(1 + \tfrac{i}{2}\tau\cdot\delta\omega \right)\psi \\ \phi &\rightarrow \phi - \delta\omega_\wedge\phi, \end{aligned} \qquad \text{A.1}$$

and the gauge transformation is,

$$\begin{aligned} \psi &\rightarrow \left(1 + ia^2\tau\cdot\phi_\wedge\delta\zeta \right)\psi \\ \phi &\rightarrow \phi + \delta\zeta + a^2[\ 2\phi\delta\zeta\cdot\phi - \delta\zeta\phi^2\ ] \end{aligned} \qquad \text{A.2}$$

where $a = (f_o/m_\pi)$, ψ and ϕ are nucleon and pion fields, ω and ζ are group parameters. They are all iso-vectors. $\wedge$ and $\cdot$ denote vector and scalar product in isospin space.

The combined transformation is,

$$\begin{aligned} \psi &\rightarrow \left(1 + \tfrac{i}{2}\tau\cdot\delta\omega + ia^2\tau\cdot\phi_\wedge\delta\zeta \right) \\ \phi &\rightarrow \phi - \delta\omega_\wedge\phi + \delta\zeta + a^2[\ 2\phi\delta\zeta\cdot\phi - \delta\zeta\phi^2\ ]. \end{aligned} \qquad \text{A.3}$$

Defining new parameters $\delta\omega_+$ and $\delta\omega_-$,

where

$$\delta\omega_\pm = \delta\omega \pm 2a\delta\zeta,$$

then A.3 becomes,

$$\psi \rightarrow [\ 1 + \tfrac{i}{4}(\tau + a\tau_\wedge\phi)\cdot\delta\omega_+ + \tfrac{i}{4}(\tau - a\tau_\wedge\phi)\cdot\delta\omega_-\ ]\psi$$

$$\phi \rightarrow \phi - \tfrac{1}{2}\delta\omega_{+}\wedge\phi + \tfrac{1}{4a}\delta\omega_{+} + \tfrac{a}{4}[2\phi\delta\omega_{+}\cdot\phi - \delta\omega_{+}\phi^{2}]$$
$$- \tfrac{1}{2}\delta\omega_{-}\wedge\phi - \tfrac{1}{4a}\delta\omega_{-} - \tfrac{a}{4}[2\phi\delta\omega_{-}\cdot\phi - \delta\omega_{-}\phi^{2}]. \qquad \text{A.4}$$

The group can be decomposed into right and left handed chiral groups R_+ and R_- where,

$$\psi \rightarrow R_{+}(\omega)\psi = [\, 1 + \tfrac{i}{2}(\, \tau + a\tau\wedge\phi\,)\cdot\omega \,]\psi,$$

with

$$\phi \rightarrow \phi - \omega\wedge\phi + \tfrac{1}{2a}\omega + \tfrac{a}{2}(\, 2\phi\omega\cdot\phi - \omega\phi^{2}\,),$$

and

$$\psi \rightarrow R_{-}(\omega)\psi = [\, 1 + \tfrac{i}{2}(\, \tau - a\tau\wedge\phi\,)\cdot\omega \,]\psi, \qquad \text{A.5}$$

with

$$\phi \rightarrow \phi - \omega\wedge\phi - \tfrac{1}{2a}\omega - \tfrac{a}{2}(\, 2\phi\omega\cdot\phi - \omega\phi^{2}\,).$$

The commutators of the group of transformation on ψ are evaluated as follows:

$$[\, R_{+}(\omega), R_{-}(\omega') \,]$$
$$= R_{+}(\omega)R_{-}(\omega') - R_{-}(\omega')R_{+}(\omega)$$
$$= [\, 1 + \tfrac{i}{2}(\tau + a\tau\wedge\phi')\cdot\omega \,][\, 1 + \tfrac{i}{2}(\tau - a\tau\wedge\phi)\cdot\omega' \,] \qquad \text{A.6}$$
$$-[\, 1 + \tfrac{i}{2}(\tau - a\tau\wedge\phi'')\cdot\omega' \,][\, 1 + \tfrac{i}{2}(\tau + a\tau\wedge\phi)\cdot\omega \,],$$

where $\phi' = \phi - \omega'\wedge\phi - \tfrac{1}{2a}\omega' - \tfrac{a}{2}[2\phi\omega'\cdot\phi - \omega'\phi^{2}\,],$

and $\phi'' = \phi - \omega\wedge\phi + \tfrac{1}{2a}\omega + \tfrac{a}{2}[2\phi\omega\cdot\phi - \omega\phi^{2}\,].$ A.7

This change in ϕ is due to the nonlinearity of the chiral group.

With Eq. A.7, Eq. A.6 becomes

$$[\ R_+(\omega),\ R_-(\omega')\ ]$$

$$= [\ 1 + \frac{i}{2}\tau\cdot\omega + \frac{ia}{2}\tau\wedge\{\phi - \omega'\wedge\phi - \frac{1}{2a}\omega' - \frac{a}{2}(2\phi\omega'\cdot\phi - \omega'\phi^2)\}\cdot\omega\ ]$$

$$\times [\ 1 + \frac{i}{2}(\tau - a\tau\wedge\phi)\cdot\omega'\ ]$$

$$- [1 + \frac{i}{2}\tau\cdot\omega' - \frac{ia}{2}\tau\wedge\{\phi - \omega\wedge\phi + \frac{1}{2a}\omega + \frac{a}{2}(2\phi\omega\cdot\phi - \omega\phi^2)\}\cdot\omega']$$

$$\times [\ 1 + \frac{i}{2}\tau\cdot\omega + \frac{ia}{2}(\tau\wedge\phi)\cdot\omega\ ]$$

$$= -\frac{1}{4}[i(\tau\wedge\omega')\cdot\omega - i(\tau\wedge\omega)\cdot\omega' + (\tau\cdot\omega)(\tau\cdot\omega') - (\tau\cdot\omega')(\tau\cdot\omega)]$$

$$+\frac{a}{4}[-2i\{\tau\wedge(\omega'\wedge\phi)\}\cdot\omega + (\tau\cdot\omega)\{(\tau\wedge\phi)\cdot\omega'\} - \{(\tau\wedge\phi)\cdot\omega\}(\tau\cdot\omega')$$

$$-2i\{\tau\wedge(\omega\wedge\phi)\}\cdot\omega' + (\tau\cdot\omega')\{(\tau\wedge\phi)\cdot\omega\} - \{(\tau\wedge\phi)\cdot\omega'\}(\tau\cdot\omega)]$$

$$+\frac{a^2}{4}[-i\{\tau\wedge(2\phi\omega'\cdot\phi - \omega'\phi^2)\}\cdot\omega + \{(\tau\wedge\phi)\cdot\omega\}\{(\tau\wedge\phi)\cdot\omega'\}$$

$$+i\{\tau\wedge(2\phi\omega\cdot\phi - \omega\phi^2)\}\cdot\omega' - \{(\tau\wedge\phi)\cdot\omega'\}\{(\tau\wedge\phi)\cdot\omega\}\ ]$$

using $(\tau\cdot A)(\tau\cdot B) = i\tau\cdot(A\wedge B) + A\cdot B$, where A and B are arbitrary isovectors,

$$= +\frac{a}{4}[-2i\tau\cdot\{(\omega'\wedge\phi)\wedge\omega + (\omega\wedge\phi)\wedge\omega'\}$$

$$+i\tau\cdot\{\omega\wedge(\phi\wedge\omega') + \omega'\wedge(\phi\wedge\omega)\} + \omega\cdot(\phi\wedge\omega') + \omega'\cdot(\phi\wedge\omega)$$

$$-i\tau\cdot\{(\phi\wedge\omega)\wedge\omega' + (\phi\wedge\omega')\wedge\omega\} - (\phi\wedge\omega)\cdot\omega' - (\phi\wedge\omega')\cdot\omega\ ]$$

$$+\frac{a^2}{4}[-2i\{\tau\cdot(\phi\wedge\omega)\}(\omega'\cdot\phi) + 2i\{\tau\cdot(\phi\wedge\omega')\}(\omega\cdot\phi)$$

$$+i\tau\cdot(\omega'\wedge\omega)\phi^2 - i\tau\cdot(\omega\wedge\omega')\phi^2$$

$$+i\tau\cdot\{(\phi\wedge\omega)\wedge(\phi\wedge\omega')\} - i\tau\cdot\{(\phi\wedge\omega')\wedge(\phi\wedge\omega)\}$$

$$+(\phi\wedge\omega)\cdot(\phi\wedge\omega') - (\phi\wedge\omega')\cdot(\phi\wedge\omega)\ ]$$

$$= \frac{a}{4}[(-2 + 1 + 1)i\tau\cdot(\omega'\wedge\phi)\wedge\omega + (-2 + 1 + 1)i\tau\cdot(\omega\wedge\phi)\wedge\omega'\ ]$$

$$+\frac{a^2}{4}[2i\tau\cdot\{-(\omega'\cdot\phi)(\phi\wedge\omega) + (\omega\cdot\phi)(\phi\wedge\omega') +\phi^2(\omega'\wedge\omega)$$

$$+ (\phi\wedge\omega)\wedge(\phi\wedge\omega')\}\ ]$$

$$= \frac{a^2}{4}2i\tau\cdot\{-(\omega'\cdot\phi)(\phi\wedge\omega) + (\omega\cdot\phi)(\phi\wedge\omega')$$

$$-[\omega\cdot(\phi\wedge\omega')]\phi + \phi^2(\omega'\wedge\omega)\}$$

$$= \frac{a^2}{4}2i\tau\cdot\{[(\omega\wedge\omega')\wedge\phi]\wedge\phi - (\omega'\cdot\phi)(\phi\wedge\omega) + (\omega\cdot\phi)(\phi\wedge\omega')\}$$

$$= \frac{a^2}{4}2i\tau\cdot\{(\omega\cdot\phi)(\omega'\wedge\phi) - (\omega'\cdot\phi)(\omega\wedge\phi) - (\omega'\cdot\phi)(\phi\wedge\omega)$$

$$+(\omega\cdot\phi)(\phi\wedge\omega')\}$$

$$\equiv 0.$$

Hence:

$$[\ R_+(\omega),\ R_-(\omega')\ ] \equiv 0\ .$$

Similarily,

$[\ R_{\pm}(\omega),\ R_{\pm}(\omega')\ ]$

$= [\ 1 + \frac{i}{2}(\tau \pm a\tau_{\wedge}\phi')\cdot\omega][\ 1 + \frac{i}{2}(\tau \pm a\tau_{\wedge}\phi)\cdot\omega']$

$- [\ 1 + \frac{i}{2}(\tau \pm a\tau_{\wedge}\phi'')\cdot\omega'][\ 1 + \frac{i}{2}(\tau \pm a\tau_{\wedge}\phi)\cdot\omega]$

where,

$$\phi' = \phi - \omega'_{\wedge}\phi \pm \frac{1}{2a}\omega' \pm \frac{a}{2}[2\phi\omega'\cdot\phi - \omega'\phi^2]$$

$$\phi'' = \phi - \omega_{\wedge}\phi \pm \frac{1}{2a}\omega \pm \frac{a}{2}[2\phi\omega\cdot\phi - \omega\phi^2]$$

$= [\ 1 + \frac{i}{2}\tau\cdot\omega \pm \frac{ia}{2}\tau\cdot(\phi_{\wedge}\omega) \mp \frac{ia}{2}\tau\cdot\{(\omega'_{\wedge}\phi)_{\wedge}\omega\} + \frac{i}{4}\tau\cdot(\omega'_{\wedge}\omega)$

$+ \frac{ia^2}{4}\tau\cdot\{2(\omega'\cdot\phi)(\phi_{\wedge}\omega) - \phi^2(\omega'_{\wedge}\omega)\}]$

$\times [\ 1 + \frac{i}{2}\tau\cdot\omega' \pm \frac{ia}{2}\tau\cdot(\phi_{\wedge}\omega')\ ]$

$- [\ 1 + \frac{i}{2}\tau\cdot\omega' \pm \frac{ia}{2}\tau\cdot(\phi_{\wedge}\omega') \mp \frac{ia}{2}\tau\cdot\{(\omega_{\wedge}\phi)_{\wedge}\omega'\} + \frac{i}{4}\tau\cdot(\omega_{\wedge}\omega')$

$+ \frac{ia^2}{4}\tau\cdot\{2(\omega\cdot\phi)(\phi_{\wedge}\omega') - \phi^2(\omega_{\wedge}\omega')\}]$

$\times [\ 1 + \frac{i}{2}\tau\cdot\omega \pm \frac{ia}{2}\tau\cdot(\phi_{\wedge}\omega)\ ]$

$= 1 + \frac{i}{2}\tau\cdot(\omega+\omega') \pm \frac{ia}{2}\tau\cdot[\phi_{\wedge}(\omega+\omega')] - \frac{1}{4}(\tau\cdot\omega)(\tau\cdot\omega')$

$\mp \frac{a}{4}[(\tau\cdot\omega)\{\tau\cdot(\phi_{\wedge}\omega')\} + \{\tau\cdot(\phi_{\wedge}\omega)\}(\tau\cdot\omega')]$

$- \frac{a^2}{4}\{\tau\cdot(\phi_{\wedge}\omega)\}\{\tau\cdot(\phi_{\wedge}\omega')\} \mp \frac{ia}{2}\tau\cdot\{(\omega'_{\wedge}\phi)_{\wedge}\omega\} + \frac{i}{4}\tau\cdot(\omega'_{\wedge}\omega)$

$+ \frac{ia^2}{4}\tau\cdot\{(2\phi\omega'\cdot\phi - \omega'\phi^2)_{\wedge}\omega\}$

$$- 1 - \frac{i}{2}\tau\cdot(\omega'+\omega) \mp \frac{ia}{2}\tau\cdot[\phi\wedge(\omega'+\omega)] + \frac{1}{4}(\tau\cdot\omega')(\tau\cdot\omega)$$

$$\pm \frac{a}{4}[(\tau\cdot\omega')\{\tau\cdot(\phi\wedge\omega)\} + \{\tau\cdot(\phi\wedge\omega')\}(\tau\cdot\omega)]$$

$$+ \frac{a^2}{4}\{\tau\cdot(\phi\wedge\omega')\}\{\tau\cdot(\phi\wedge\omega)\} \pm \frac{ia}{2}\tau\cdot\{(\omega\wedge\phi)\wedge\omega'\} - \frac{i}{4}\tau\cdot(\omega\wedge\omega')$$

$$- \frac{ia^2}{4}\tau\cdot\{(2\phi\omega\cdot\phi - \omega\phi^2)\wedge\omega\}$$

$$= i\tau\cdot(\omega'\wedge\omega) \mp ia\tau\cdot[\omega\wedge(\phi\wedge\omega') - \omega'\wedge(\phi\wedge\omega)]$$

$$-i\frac{a^2}{2}\tau\cdot[(\phi\wedge\omega)\wedge(\phi\wedge\omega') - (\omega'\cdot\phi)(\phi\wedge\omega) + (\omega\cdot\phi)(\phi\wedge\omega') + (\omega'\wedge\omega)\phi^2]$$

$$= i\tau\cdot(\omega'\wedge\omega) \pm ia\tau\cdot[\phi\wedge(\omega'\wedge\omega)]$$

$$= i(\ \tau \pm a\tau\wedge\phi\ )\cdot(\ \omega'\wedge\omega\ ).$$

Hence:

$$[\ R_{\pm}(\omega),\ R_{\pm}(\omega')\ ] = R_{\pm}(\omega'\wedge\omega) - 1.$$

These commutators, together with Eq. (A.8) prove the stated group structure $O(4) = SU(2)\times SU(2)$, of the transformations on ψ. The gruop structure of the transformations on ϕ has been shown by Schwinger.[21]

APPENDIX B CP PARITY OF THE CHIRAL CURRENTS

Applying the CP operator on the vector and axial vector currents, equations (15), (17) and (16), (18) gives the following results;

$$V_\mu(\pi^\pm) \xrightarrow{CP} \pm\, CP\, \pi^0 i\overleftrightarrow{\partial}_\mu \pi^\pm\, (CP)^{-1}$$

$$= \pm\, \varepsilon_\mu \pi^0 i\overleftrightarrow{\partial}_\mu \pi^\mp$$

$$= -\,\varepsilon_\mu V_\mu(\pi^\mp), \qquad (B.1)$$

$$V_\mu(\pi^0) \xrightarrow{CP} CP\, \pi^+ i\overleftrightarrow{\partial}_\mu \pi^-\, (CP)^{-1} = \varepsilon_\mu \pi^- i\partial_\mu \pi^+$$

$$= -\,\varepsilon_\mu V_\mu(\pi^0), \qquad (B.2)$$

$$V_\mu(K^+) \xrightarrow{CP} CP\, \frac{i}{2}\left(\frac{f_K}{f_\pi}\right)\{\partial_\mu(\pi^0K^+ + \sqrt{2}\pi^+K^0) - g(\partial_\mu\pi^0K^+ + \sqrt{2}\partial_\mu\pi^-K^0)\}\, (CP)^{-1}$$

$$= \varepsilon_\mu e^{i\alpha}\left(\frac{f_K}{f_\pi}\right)\frac{i}{2}\{\partial_\mu(\pi^0K^- + \sqrt{2}\pi^-\overline{K}^0) - g(\partial_\mu\pi^0K^- + \sqrt{2}\partial_\mu\pi^-\overline{K}^0)\}$$

$$= -\varepsilon_\mu e^{i\alpha} V_\mu(K^-),$$

$$(B.3)$$

$$V_\mu(K^-) \xrightarrow{CP} -\varepsilon_\mu e^{-i\alpha} V_\mu(K^+),$$

$$V_\mu(K^0) \xrightarrow{CP} -\varepsilon_\mu e^{i\alpha} V_\mu(\overline{K}^0),$$

$$(B.4)$$

$$V_\mu(\overline{K}^0) \xrightarrow{CP} -\varepsilon_\mu e^{-i\alpha} V_\mu(K^0),$$

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$$A_\mu(\pi^\pm) \xrightarrow{CP} -\varepsilon_\mu\left(f_\pi \partial_\mu \pi^\mp + (4f_\pi^2)^{-1}\{2(\pi\cdot\partial_\mu\pi)\pi^\mp - 3\pi^2\partial_\mu\pi^\mp\}\right)$$

$$= -\varepsilon_\mu A_\mu(\pi^\mp), \tag{B.5}$$

$$A_\mu(\pi^0) \xrightarrow{CP} -\varepsilon_\mu A_\mu(\pi^0), \tag{B.6}$$

$$A_\mu(K^+) \xrightarrow{CP} -e^{i\alpha}\varepsilon_\mu\{f_K\partial_\mu K^- - (f_K/8f_\pi^2)\partial_\mu(\pi^2K^-) + \cdot\cdot\}$$

$$= -\varepsilon_\mu e^{i\alpha} A_\mu(K^-), \tag{B.7}$$

$$A_\mu(K^-) \xrightarrow{CP} -\varepsilon_\mu e^{-i\alpha} A_\mu(K^+),$$

$$A_\mu(K^0) \xrightarrow{CP} -\varepsilon_\mu e^{i\alpha} A_\mu(\bar{K}^0),$$

$$A_\mu(\bar{K}^0) \xrightarrow{CP} -\varepsilon_\mu e^{-i\alpha} A_\mu(K^0), \tag{B.8}$$

where $\varepsilon_4 = 1$, $\varepsilon_{1,2,3} = -1$, is the factor arrising from the spatial reversion of the differential operator, α is an arbitrary phase factor.

In the above operations, we have used the fact that the charge conjugation operator converts a field corresponding to particle into that corresponding to an antiparticle. Since we are dealing with pseudoscalar fields, a negative sign appears on each field under parity operation. Also since the strong interaction conserve strangeness, we could define the C operation on strange fields as,

$$C\,(K^+,K^0)\,C^{-1} = e^{i\alpha}(K^-,\bar{K}^0) \text{ and } C\,(K^-,\bar{K}^0)\,C^{-1} = e^{-i\alpha}(K^+,\bar{K}^0).$$

Collecting the results from equations (B.1) to (B.8), we can thus write,

$$
\begin{aligned}
& CP\ J_\mu(\Delta Y, \Delta I_3)\ (CP)^{-1} \\
&= CP\ \{V_\mu(\Delta Y, \Delta I_3) + A_\mu(\Delta Y, \Delta I_3)\}\ (CP)^{-1} \\
&= -\ \varepsilon_\mu e^{i\alpha\Delta Y}\{V_\mu(-\Delta Y, -\Delta I_3) + A_\mu(-\Delta Y, -\Delta I_3)\} \\
&= -\ \varepsilon_\mu e^{i\alpha\Delta Y}\ J_\mu(-\Delta Y, -\Delta I_3).
\end{aligned} \tag{B.9}
$$

A more general way of discussing CP parity of the chiral currents is by making use of the property that the strong interaction Lagrangain L, and so its variation δL, is invariant under the CP operation. Thus, the CP parity of the V_μ and A_μ, as defined in Eq.(10) of chapter III, must be the same as that of $\partial_\mu \delta\omega$ and $\partial_\mu \delta\zeta$. Since $\delta\zeta$ is a displacement of the meson field, it has the same CP parity as the meson field. And through equation (8), the CP parity of $\delta\omega$ is determined. The CP parity of the chiral currents determined in this way is identical to equation (B.9).

APPENDIX C CONTRIBUTION FROM THE TERM $J_\mu(K^0)J_\mu(\eta^0)$ TO KAON TO TWO- AND THREE-PION DECAY

The currents $V_\mu(\eta^0)$ and $A_\mu(\eta^0)$ written in terms of the kaon, pion and η^0-meson fields, K, π and η^0, are;

$$V_\mu(\eta^0) = -i(\sqrt{6}/2)\{K^-\overleftrightarrow{\partial}_\mu K^+ + \bar{K}^0\overleftrightarrow{\partial}_\mu K + (2/3)\eta^0\overleftrightarrow{\partial}_\mu\eta^0\},$$

and

$$\begin{aligned}A_\mu(\eta^0) = f\partial_\mu\eta^0 - \frac{\sqrt{6}}{f}\{&(1/\sqrt{2})(\partial_\mu\pi^0)(K^-K^+ - \bar{K}^0K^0) + (\partial_\mu\pi^+)K^-K^0 \\ &+ (\partial_\mu\pi^-)\bar{K}^0K^+ - (2/\sqrt{6})\eta^0(\partial_\mu K^+K^- + \partial_\mu K^-K^+) \\ &- (2/\sqrt{6})\eta^0(\partial_\mu\bar{K}^0K^0 + \partial_\mu K^0\bar{K}^0) \\ &- (4/3\sqrt{6})(\partial_\mu\eta^0)\eta^0\eta^0\ \} \\ - \frac{a_3\sqrt{6}}{4f}\partial_\mu\{&(1/\sqrt{2})\pi^0(K^+K^- - \bar{K}^0K^0) + \pi^+K^-K^0 \\ &+ \pi^-K^+\bar{K}^0 - (4/3\sqrt{6})\eta^0\eta^0\eta^0 \\ &- (3/\sqrt{6})\eta^0(K^+K^- + \bar{K}^0K^0)\ \}\ .\end{aligned}$$

The product $J_\mu(K^0)J_\mu(\eta^0)$, where $J_\mu(K^0)$ is given by Eq.(17) and (18) in chapter II, contains no matrix element, to the lowest order in f, for the kaon to two- or three-pion decay. To higher order in f, processes such as,

$$\begin{array}{llll} K & \rightarrow & K & + \ \pi \\ & & \hookrightarrow & \quad \pi \end{array}$$

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and

$$\begin{tikzcd} K \arrow[r] & K \arrow[drr] & + & \pi \\ & & & \pi + \pi \end{tikzcd}$$

could occur, but the phase space factors of these processes are practically zero. So these contribution to the $K \to 2\pi$ and $K \to 3\pi$ processes can be neglected.

APPENDIX D OTHER WAY OF OBTAINING Eq.(30)

According to the formulism of Wu and Yang , the states K_S and K_L , assuming CPT invariance,are defined as;

$$K_S = N(K^0 + e^{i\phi}\overline{K}^0)$$

$$K_L = N(K^0 - e^{i\phi}\overline{K}^0)$$

where N is a normalization constant and ϕ a complex number equal to -2α is our notation.

The angle ϕ is related to the off-diagonal elements of the mass and decay matrix , M and Γ , through the following equations;

$$\tan\phi = E_2/E_1,$$

where,

$$iE_1 + E_2 = \Gamma_{12} + iM_{12},$$

and

$$iE_1 - E_2 = \Gamma_{21} + iM_{21}.$$

Since $M_{21}(\Gamma_{21})$ is the hermitian conjugate of $M_{12}(\Gamma_{12})$, we have,

$$E_1 = -i(\mathrm{Re}\Gamma_{12} + i\mathrm{Re}M_{12}),$$

and

$$E_2 = i(\mathrm{Im}\Gamma_{12} + i\mathrm{Im}M_{12}).$$

The mass and decay matrix has contributions from 2π intermediate states in the $I = 0$ and $I = 2$ states,

from 3π intermediate states, and from leptonic states, etc. The mass matrix involves off the energy-shell contributions but the decay matrix involves only the contributions on the energy-shell. Because of the phase space factor, the on-shell terms involving two particles will always be considerably larger than three- or four-particle terms, we assume that this is also true for the off-shell terms. Thus only the 2π states give significant contribution to the mass and decay matrix. We further assume that the current-current form of lagrangian and the form of the currents are even valid off the energy shell, and we ignore the possibility of resonance which might alter the amplitude for the 3/2 and 1/2 transition. We then have;

$$\Gamma_{12} = 2\pi \sum_{I=0,2} \rho_I(M)\langle K^0|L|2\pi_I\rangle\langle 2\pi_I|L|\bar{K}^0\rangle,$$

and

$$M_{12} = \sum_{I=0,2} P\int \frac{dE_\pi}{2E_\pi - M_K}\rho_I(E_\pi)\langle K^0|L|2\pi_I\rangle\langle 2\pi_I|L|\bar{K}^0\rangle,$$

where P denotes the princ..p.. value and $\rho_I(M)$, $\rho_I(E_\pi)$ stands for the two pion phase space factor in the isospin I state, the (M) and (E_π) denote on and off the energy shell terms.

The K to 2π transition matrix element is given below;

$$\langle 2\pi_{I=0}|L_{1/2}|K^0\rangle = \sqrt{6}iDG_{1/2}e^{i\beta_0},$$

$$\langle 2\pi_{I=2}|L_{3/2}|K^0\rangle = \sqrt{2}\sqrt{6}iDG_{3/2}e^{i\beta_2},$$

where, $D = if_K g(M_K^2 - 2m_\pi^2)/4$ on the mass shell,

and $D = if_K g(4E_\pi^2 - 2m_\pi^2)/4$ off the mass shell.

Hence we have,

$$\Gamma_{12} = 2\pi(6\rho_0 G_{1/2}^2 e^{-2i\beta_0} + 12\rho_2 G_{3/2}^2 e^{-2i\beta_2})D^2,$$

and

$$M_{12} = P\int \frac{dE_\pi}{2E_\pi - M_K}(6\rho_0 G_{1/2}^2 e^{-2i\beta_0} + 12\rho_2 G_{3/2}^2 e^{-2i\beta_2})D^2.$$

If one assumes that $\rho_0(M)/\rho_2(M) = \rho_0(E_\pi)/\rho_2(E_\pi)$, then the phase of M_{12} equals to that of Γ_{12}. With Eq. (D.2), (D.3) and (D.6) we have,

$$\begin{aligned}
\tan\phi &= -\tan 2\alpha \\
&= \frac{i(\mathrm{Im}\Gamma_{12} + i\mathrm{Im}M_{12})}{-i(\mathrm{Re}\Gamma_{12} + i\mathrm{Re}M_{12})} \\
&= -\text{phase of } \Gamma_{12} \\
&= -\text{phase of } M_{12} \\
&= -\frac{\sin 2\beta_0 + 2r\sin 2\beta_2}{\cos 2\beta_0 + 2r\cos 2\beta_2}\,.
\end{aligned}$$

Thus we obtain

$$\sin 2(\beta_0+\alpha) = -r^2\sin 2(\beta_2+\alpha),$$

which is identical to Eq. (30) in chapter II.